

Quantum dense coding scheme via cavity decay

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We investigate a secure scheme for implementing quantum dense coding via cavity decay and liner optics devices. Our scheme combines two distinct advantages: atomic qubit serves as stationary bit and photonic qubit as flying bit, thus it is suitable for long distant quantum communication.

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I. INTRODUCTION

Quantum entanglement, a fundamental feature of many-body quantum mechanical systems, was regarded as a key resource for many tasks in quantum information processing [1, 2, 3]. Quantum dense coding (QDC) is a process to send two classical bits (cbits) of information from a sender (Alice) to a remote receiver (Bob) by sending only a single qubit. It works in the following way. Initially, Alice and Bob shared a maximally entangled state. The first step is an encoding process where Alice performs one of the four local operations on her qubit. Then she sends the qubit to Bob. The last step is a decoding process. After Bob received the qubit, he can discriminate the local operation of Alice by using only local operations, *i.e.*, Bell state measurement in the work of Bennett *et al.* [1]. Later, due to its predominate importance in quantum communication, QDC attracts many public attentions. Barenco and Ekert [4] first addressed the question of QDC with partially entangled state, and they focused on deterministic QDC and considered the classical capacity of it. Conversely, one can also consider the case of QDC by partial entangled state with maximal classical capacity (2 cbits per qubit) in a non-deterministic or probabilistic way [5]. On the other hand, QDC has been experimentally demonstrated using optical systems [6], nuclear magnetic resonance (NMR) techniques [7] and trapped ions systems [8]. But, in the conventional process of QDC the receiver can always successfully cheat Alice if he want and use the information willingly. Now, the question arises, is there any secure QDC scheme exist? Fortunately, we note that quantum secret sharing (QSS) [3] is likely to help in protecting secret information. Here, in analogy with QSS, we term *secure QDC* as a process securely distributing information via QDC among many parties in a way only when they cooperate can they read the distributed secret information.

In the realm of atom, cavity quantum electrodynamics (QED) techniques has been proved to be a promising candidate for the physical realization of quantum information processing [9]. The cavities usually act as memories, thus the decoherence of the cavity field becomes one of the main obstacles for the implementation of quantum information in cavity QED. Recently, Zheng and Guo proposed a novel scheme

[10], which greatly prolong the efficient decoherence time of the cavity with a virtually excited nonresonant cavity. Os-naghi *et al.* [11] had experimentally implemented the scheme using two Rydberg atoms crossing a nonresonant cavity. Following these progresses, schemes for implementing QDC are also proposed [12], where atomic qubit is used as both stationary and flying bit. But, it is well known that atomic qubits are only ideal stationary qubits, not suitable for long distance transmission. Thus the realization of long distance quantum communication with atomic qubits serving as flying qubits as suggested in [12] is an difficult experimental challenge. Meanwhile, spontaneous and detected decay are unavoidable in practical quantum information processing in cavity QED system [13]. However, it is shown recently that the detection (or the non detection) of decays can be used to entangle the states of distinct atoms [14]. Furthermore, it can be used for quantum communication protocols such as QT [15, 16], as Photonic qubits are perfect candidate for flying qubits.

Here, we investigate a physical scheme for implementing QDC via cavity decay and liner optics devices. The motivation of this work is twofold: investigating physical QDC scheme via cavity QED technology and solving the problem of security in practical QDC schemes. Our scheme combines two distinct advantages: atomic qubit serves as stationary qubit and photonic qubit as flying qubit, thus it is suitable for long distance quantum communication. We investigate the scheme in a way similar to QSS [3] via a tripartite entangled GHZ state, thus it is also a secure one. We firstly consider the atom-cavity interaction in section 2, then provide our scheme in section 3. Section 4 is some discussions about our scheme and summary of our paper.

II. ATOM-CAVITY INTERACTION

Here and afterwards, each of the atoms has a three-level structure, which has two ground states $|g\rangle, |e\rangle$ (*e.g.* hyperfine ground states) and an excited state $|r\rangle$ as depicted in Fig.(1). It is an adiabatic evolution for the $|e\rangle \rightarrow |r\rangle$ transition, which is driven by a classical laser pulse with coupling coefficient Ω . The $|r\rangle \rightarrow |g\rangle$ transition is driven by the quantized cavity mode with coupling coefficient g . Both the classical laser pulse and the cavity mode are detuned from their respective transition frequencies by the same amount Δ . Assuming the atom is trapped in a specific position in the cavity, and the coupling coefficients Ω and g are constant during the interaction. In the case of $\Omega g / \Delta^2 \ll 1$, the upper level $|r\rangle$ can be de-

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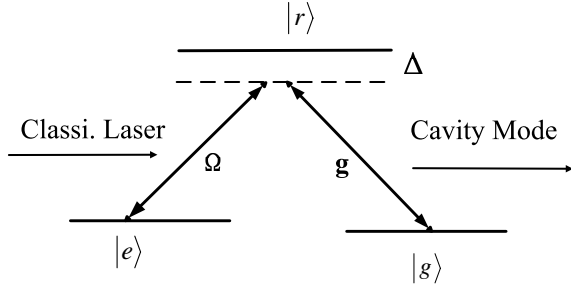


FIG. 1: Atom level structure in our scheme. The $|e\rangle \rightarrow |r\rangle$ transition is driven by a classical laser pulse with coupling Ω , and the $|r\rangle \rightarrow |g\rangle$ transition is driven by the quantized cavity mode with coupling g . Both the classical laser pulse and the cavity mode are detuned from their respective transition frequencies by a same amount Δ .

coupled from the evolution. When $\Delta \gg \gamma$ the spontaneous decay rate γ from $|r\rangle$ level can be neglected [15]. Suppose $\Omega = g$, the effective Hamiltonian, in the interaction picture, is

$$H_e = i\delta(a|e\rangle\langle g| - a^\dagger|g\rangle\langle e|) - ika^\dagger a, \quad (1)$$

where $\delta = g\Omega/\Delta$, a and a^\dagger are the annihilation and creation operators of the cavity mode, k is the photon decay rate from the cavity. The time evolution of the interaction under the Hamiltonian in Eq. (1) are

$$|g\rangle|0\rangle \rightarrow |g\rangle|0\rangle, \quad (2a)$$

$$|e\rangle|0\rangle \rightarrow (\alpha|e\rangle|0\rangle + \beta|g\rangle|1\rangle). \quad (2b)$$

where we have discarded the phase factor, which can be removed by a simple rotate operation. The coefficients in Eq. (2b) are

$$\alpha = e^{-\frac{1}{2}kt} \left(\cos \frac{1}{2}\Omega_k t + \frac{\Omega}{\Omega_k} \sin \frac{1}{2}\Omega_k t \right),$$

$$\beta = -\frac{2\Omega}{\Omega_k} e^{-\frac{1}{2}kt} \sin \frac{1}{2}\Omega_k t$$

with $\Omega_k = \sqrt{4\delta^2 - k^2}$.

III. SECURE QDC WITH TRIPARTITE GHZ STATE

Suppose Alice wants to send secret information to a distant receiver Bob. As she does not know whether he is honest or not, she makes the information shared by two users, *i.e.*, Bob and Charlie. If and only if they collaborate, one of the users can read the information, furthermore, individual users could not do any damage to the process. The sender can probabilistically transmit two cbits of information by sending only one qubit to the two receivers. By collaboration, one of them could obtain the exact information, furthermore, any attempt to obtain the secret information without cooperation cannot

succeed in a deterministic way. Assume the three parties, *i.e.* Alice, Bob and Charlie, initially share a tripartite entangled state, which has been prepared [17] in the GHZ type entangled state

$$|\psi\rangle_{1,2,3} = \frac{1}{\sqrt{2}}(|eee\rangle + |ggg\rangle)_{1,2,3}, \quad (3)$$

where $|e\rangle$ and $|g\rangle$ are the excited and ground states of the atoms, respectively. Atoms 1, 2 and 3 belong to Alice, Bob and Charlie, respectively.

Step 1. Alice decides to select one of the following two possible choices. With probability p Alice selects the first choice of security checking, which aims to check the security of quantum channel, and then the procedure continues to Step 2. Otherwise, Alice can also move to the information encoding step with probability $(1 - p)$, the aim of which is to encode and implement the QDC procedure. In this case, the procedure goes to Step 3.

Step 2. Security checking. Hillery *et al.* [3] show that tripartite entangled GHZ state is sufficient to detect a potential eavesdropper in the channel. In other words, the eavesdropper could not succeed in a deterministic way during the QDC procedure.

Step 3. Information encoding. Alice performs one of the four local operations $\{I, \sigma^x, i\sigma^y, \sigma^z\}$ on her atom. These operations denote 2 cbits information, and will transform the state (3) to

$$|\psi\rangle_{1,2,3} = \frac{1}{\sqrt{2}}(|eee\rangle + |ggg\rangle)_{1,2,3}, \quad (4a)$$

$$|\psi\rangle_{1,2,3} = \frac{1}{\sqrt{2}}(|gee\rangle + |egg\rangle)_{1,2,3}, \quad (4b)$$

$$|\psi\rangle_{1,2,3} = \frac{1}{\sqrt{2}}(|gee\rangle - |egg\rangle)_{1,2,3}, \quad (4c)$$

$$|\psi\rangle_{1,2,3} = \frac{1}{\sqrt{2}}(|eee\rangle - |ggg\rangle)_{1,2,3}. \quad (4d)$$

Now the information is encoded into the pure entangled state, which is shared among the three parties, and the encoding of the two cbits information is completed.

Step 4. Information extracting. Alice applies a classical laser pulse on atom to switch on the effective Hamiltonian H_e of atom 1 and cavity A, and then lead the photonic qubit flying to one of the two receivers (*i.e.*, Bob). We will later discuss which is the party Alice sends her qubit to is not arbitrary. After a party receives the qubit, he will have a higher probability of successful cheat compared with the one who have not in the QDC procedure. So, Alice would send her atom to the party, which is less likely to cheat. We will discuss this latter in detail. Assume Bob was selected to receive Alice's photonic qubit, and both cavities are initially prepared in vacuum state. The setup of our scheme is shown in figure (2).

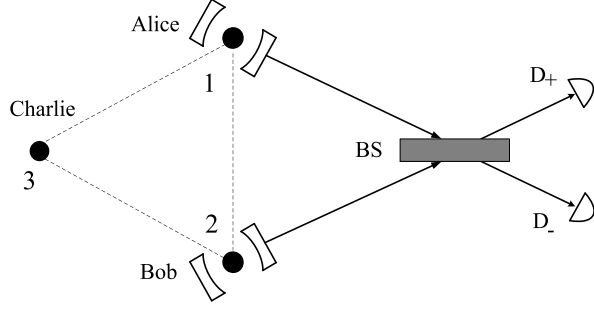


FIG. 2: The setup is adapted to implement the secure QDC scheme. Atoms 1, 2 and 3 are initially prepared in tripartite entangled state. Label the cavity of Alice and Bob as A and B , respectively, and they were both initially prepared in the vacuum state. The 50/50 beam splitter (BS) and single-photon detectors D_{\pm} are located in Bob's side.

Meanwhile, Bob also switch on the effective Hamiltonian of atom 2 and cavity B. An interaction time of $\tan \frac{\Omega_k t}{2} = -\frac{\Omega_k}{k}$ for both system leads the state (4) to

$$|\psi\rangle_{A,B,3} = \frac{1}{\sqrt{2}}(\beta^2|11\rangle_{A,B}|e\rangle_3 + |00\rangle_{A,B}|g\rangle_3), \quad (5a)$$

$$|\psi\rangle_{A,B,3} = \frac{\beta}{\sqrt{2}}(|01\rangle_{A,B}|e\rangle_3 + |10\rangle_{A,B}|g\rangle_3), \quad (5b)$$

$$|\psi\rangle_{A,B,3} = \frac{\beta}{\sqrt{2}}(|01\rangle_{A,B}|e\rangle_3 - |10\rangle_{A,B}|g\rangle_3), \quad (5c)$$

$$|\psi\rangle_{A,B,3} = \frac{1}{\sqrt{2}}(\beta^2|11\rangle_{A,B}|e\rangle_3 - |00\rangle_{A,B}|g\rangle_3). \quad (5d)$$

where the subscript of A and B corresponds to the cavity of Alice and Bob, respectively. We have omitted the state of atoms 1 and 2 in Eq. (5) as they disentangled with the subsystem of $(A,B,3)$. After the controlled interaction, which can be controlled easily by a velocity selector [9], the states of atoms 1 and 2 are both in their ground states.

Charlie let his atom crosses a classical field tuned to the transitions $|g\rangle \leftrightarrow |e\rangle$. Choose the amplitudes and phases of the classical fields appropriately so that atom 3 undergoes the following transitions

$$|e\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle_3 + |g\rangle_3), \quad (6a)$$

$$|g\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle_3 - |g\rangle_3), \quad (6b)$$

which leads the state in Eq. (5) into

$$|\psi\rangle_{A,B,3} = \sqrt{\frac{\beta^4 + 1}{2}}(|\phi^+\rangle_{A,B}|e\rangle_3 + |\phi^-\rangle_{A,B}|g\rangle_3), \quad (7a)$$

$$|\psi\rangle_{A,B,3} = \frac{\beta}{\sqrt{2}}(|\psi^+\rangle_{A,B}|e\rangle_3 + |\psi^-\rangle_{A,B}|g\rangle_3), \quad (7b)$$

$$|\psi\rangle_{A,B,3} = \frac{\beta}{\sqrt{2}}(|\psi^-\rangle_{A,B}|e\rangle_3 + |\psi^+\rangle_{A,B}|g\rangle_3), \quad (7c)$$

$$|\psi\rangle_{A,B,3} = \sqrt{\frac{\beta^4 + 1}{2}}(|\phi^-\rangle_{A,B}|e\rangle_3 + |\phi^+\rangle_{A,B}|g\rangle_3). \quad (7d)$$

where

$$|\psi^{\pm}\rangle_{A,B} = \frac{1}{\sqrt{2}}(|01\rangle_{A,B} \pm |10\rangle_{A,B}), \quad (8a)$$

$$|\phi^{\pm}\rangle_{A,B} = \frac{1}{\sqrt{\beta^4 + 1}}(\beta^2|11\rangle_{A,B} \pm |00\rangle_{A,B}). \quad (8b)$$

Obviously, one can see that there is an explicit correspondence between Alice's operation and the measurements results of the two receivers, which means that if they cooperate, both of them can read the information. But if they do not choose to cooperate, neither of the two users could obtain the information by local operation in a deterministic manner. In this way, we complete the procedure of secret extraction, and the above procedures from step 1 to step 4 constitute a complete process of secure QDC. One can repeat the above procedures until all the information was sent.

Then the only task is to discriminate the four state in Eq. (8). We note that the two states in Eq. (8a) can be easily discriminated [18] from Eq. (8), and the implementation is shown in figure (1). The discrimination of Eq. (8b) from Eq. (8) requires photon number discrimination detector, which is a technique still under extensive exploration. So, our scheme is a probabilistic one.

Next, we consider the discrimination of Eq. (8a) from Eq. (8). They are discriminated by the different clicks of the two single-photon detectors as shown in figure (1). Before one of the two detectors clicks, the state (8a) will evolve to [19]

$$|\psi(t)^+\rangle_{A,B} = \frac{1}{\sqrt{2}}e^{-kt}(|01\rangle_{A,B} + |10\rangle_{A,B}). \quad (9a)$$

$$|\psi(t)^-\rangle_{A,B} = \frac{1}{\sqrt{2}}e^{-kt}(|01\rangle_{A,B} - |10\rangle_{A,B}). \quad (9b)$$

While one of the detectors D_{\pm} clicks, it corresponds to the action of the jump operators $1/\sqrt{2}(a_A \pm a_B)$ on the joint state $|\psi(t)^{\pm}\rangle$. The click of D_+ and D_- correspond to the states of (9a) and (9b), respectively. Based on the above analysis, the total probability of successfully discriminate the four states in Eq. (8) is $\beta^2 e^{-2kt}$, in other words, our scheme succeed with a probability of $\beta^2 e^{-2kt}$.

IV. DISCUSSIONS AND SUMMARY

Now, let's turn to the case if they do not choose to cooperate with each other. Without the cooperation of Charlie, Bob knows Alice's operation belongs to one of $\{\sigma^x, i\sigma^y\}$ with unit probability when the detector clicks. But he cannot further discriminate which one Alice's operation is. In other words, without the cooperation of Bob, Charlie knows nothing further about Alice's operation. If Charlie lies to Bob, Bob also has a probability of $\frac{1}{2}$ to get the correct information, so the successful cheat probability of Charlie is $\frac{1}{2}$. Conversely, Charlie only has a probability of $\frac{1}{4}$ to get the correct information, so the successful probability of Bob is $\frac{3}{4}$. This is the point that we have mentioned in the beginning of Step 4, that is, he who received Alice's atom has a higher probability of successful cheat compared with the party who have not (see Eq. (7)).

We also note the scheme can generalize to multipartite case provided Alice possesses a multipartite entangled state. Suppose she has a $(N+1)$ -qubit entangled state, qubits 2, 3, \dots $(N+1)$ are sent to N users, respectively. After she confirms that each of the users have received a qubit, she then operates one of the four local measurements on qubit 1. After that, the two cbits information was encoded into the $(N+1)$ -qubit entangled state. Later, she sends her photonic qubit to one of the rest N users. Again, he who received the photonic qubit will have a higher probability of successful cheat compared with the rest $(N-1)$ users. Only with the cooperation of all the rest users, one can obtain Alice's information. In this way, we set up a multipartite secure QDC procedure.

Next, we will discuss the experimental feasibility of the current scheme. Among the variety of systems being explored for hardware implementations for quantum communication, cavity QED system is favored because of its demonstrated advantage when subjected to coherent manipulations. The interaction time can be perfectly controlled by a velocity selector [9]. Relaxation rates of the system are small and well understood. The strong-coupling conditions are readily fulfilled [20]. The time constants involved are long enough to realize all the involved manipulations. Finally, the quantum systems are separated by centimeterscale distances, thus can be individually addressed. In our paper, we assume that the photon detector is a perfect one and no dark counts. If the finite quantum efficiency and the dark counts of the detector is taken into consideration, the fidelity and probability of

success will decrease. For an ordinary photon detector, the dark counts will reduce the fidelity of the current scheme on the order of 5% \sim 10%. If we make use of Rydberg atoms with principal quantum numbers 50 and 51, the atomic radiative time can reach $t_r = 3.0 \times 10^{-2}$ s [21]. Within the current technology, the quality factor of a cavity can reach $Q = 3.0 \times 10^8$ [9, 21], thus the effective cavity decay time can reach $t_d = 3.0 \times 10^{-3}$ s [10]. Here the coupling constant Ω and g can be carefully chosen to reach 10MHz, and the frequency detune amount can reach 100MHz [15], the requirements $\Omega g / \Delta^2 \ll 1$, $\Delta \gg \gamma$ and $\Omega_k \gg k$ can be satisfied. Using the above mentioned typical coefficients, we get that the time for the entanglement transfer is $t_1 = 1.0 \times 10^{-4}$ s. If we set the time interval for the detection stage to satisfy $t_2 = 5.0 \times 10^{-5}$ s, the time required to complete the protocol is on the order of 1.0×10^{-4} s. In addition, the finite disentanglement time for an atomic entangled state is $T_d = 1.6 \times 10^{-2}$ s [22]. So, the time required to complete the process is much shorter than the atomic radiative time, the effective cavity decay time and the finite disentanglement time for an atomic entangled state. Thus the current scheme might be realizable with the current cavity QED technology.

In summary, we have investigated a scheme for QDC with GHZ type entangled state via cavity decay and linear optics devices. The scheme is probabilistic but secure one. If and only if they cooperate with each other, they can read Alice's operational information. Any attempt to get complete information without the cooperation of the third party cannot be succeed in a deterministic way. Our scheme combines two distinct advantages: atomic qubit serves as stationary bit and photonic qubit as flying bit, thus it is suitable for long distance quantum communication. In the scheme, we implemented all the operation and they were all within current techniques, thus our suggestion may offer a simple and easy way of demonstrating secure QDC experimentally.

Acknowledgments

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